

The electrochemical behavior of a system with a limited number of molecules

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Abstract In this paper, based on Einstein relationship between diffusion and random walk, the electrochemical behavior of a system with a limited number of molecules was simulated and explored theoretically. The transition of the current vs time responses from discrete to continuous was clearly obtained as the number of redox molecules increased from 10 to 10^6 . By correlation analysis between the simulation results and the results of analytical expressions, a quantized extent parameter was proposed to investigate the underlying rules of these discrete signals, which looked stochastic. The results revealed that this parameter would be useful to describe such systems.

Keywords Discrete electron-transfer events · Single-molecule studies · Electrochemical detection · Random-walk algorithm · Theoretical simulation

Introduction

Chemical analysis is tending to a more and more small domain. A good number of techniques including optical methods, chemical methods, and scanning microscopy approaches have been employed to achieve detection toward the single-molecule level [1–5]. Among these molecule

detection techniques, electrochemical methods have gained an increasing attention because these provide a direct measurement of electron transfer process. Single or a few molecules have been trapped and detected by using nanoelectrodes or scanning electrochemical microscopy (SECM) methods, although some kind of accumulative averaging effect was involved—temporal or spatial [6–13]. Furthermore, electrochemical techniques are powerful tools in investigating interfacial electron transfer events [14–19], which now extend to bioelectrochemical systems including metalloproteins, DNA, peptides, and others [20–22]. As with the improvement in the fabrication of nanoelectrodes [23–29] and the sensitivity and measuring rate of electrochemical instruments [30, 31], one will have a good reason for the detecting of single-electron transfer event experimentally in the near future. The quantized appearances and other anomalous properties of single or a few molecules would be revealed more clearly.

However, even with those achievements we still know little about the transition process of electrochemical behaviors of the systems from micron to nanometer and single-molecule scales, i.e., from statistical to discrete, which must be important in the following single-molecule detection experiments and its further practical application such as making single-electron devices [32, 33]. By virtue of computer simulation, Amatore and coworkers gave a sketch of the transition from stochastic to statistical behavior by simulating electron hopping process with a different number of dendrimer molecules adsorbed onto a platinum ultra-microelectrode [34], which showed us a possibility to explore the properties and behaviors of a few molecules theoretically before achieving these experimentally in electrochemistry community. It is known that the macrosystem's character can be interpreted by statistical theories; however, some systems, especially those composed of a few individuals, mostly exhibit

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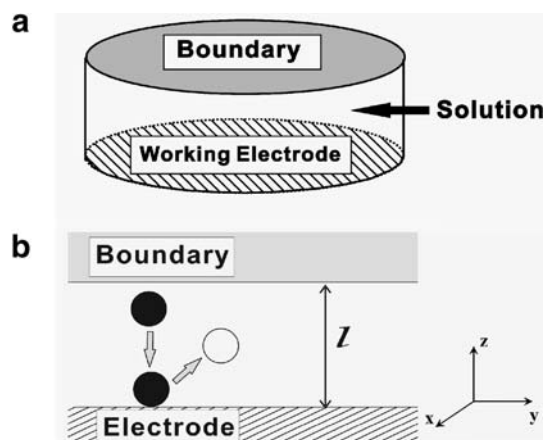


Fig. 1 **a** The three-dimensional simulation region, which was similar to a thin-layer cell. The corresponding counter electrode and the reference electrode were neglected here. **b** The illustration of an electrode process. The circle filled in black represents the molecule in oxidation state, and the circle filled in white represents the molecule in reduced state

unrepeated behavior and cannot be characterized easily. The signals from single or a few molecules were generally considered as completely stochastic and noisy. Few works about the understanding of these discrete behaviors are available now.

This paper tried to investigate the underlying rules of these discrete electrochemical behaviors. To this end, the interfacial electron transfer processes of systems with from 10 to 10^6 molecules were simulated by a random-walk algorithm. The corresponding current vs time behavior were obtained and the differences between the simulation results and the results based on statistical theory of thin-layer electrolysis were quantified, from which an identify method and parameter were proposed to characterize electrochemical systems with a limited number of molecules.

Theoretical model and simulation methods

In spite of its simplicity, random walk simulation can provide excellent descriptions of many electrochemical processes and have been widely used in the electrochemistry community [35–38]. A detailed description of this algorithm can be found in Nagy's work [37]. The main principle of this simulation was similar to previous work.

The electrochemical system chosen in this report is shown in Fig. 1. The specific electroactive molecules were dispersed in this three-simulation domain as shown in Fig. 1a. The working electrode was assumed to be a plate electrode and the initial positions of the molecules were generated by a uniform random number generator so that the distribution of the redox molecules in solution was uniform at the beginning time. In the random walk

algorithm, the molecule was allowed to take a step δ either forward or backward randomly in each x , y , and z directions during one fixed time step τ . In this simulation, the collision to the space boundary was assumed to be elastic and the system was treated as enclosed and without any mass exchange with environment, so both the motion in either x or y direction (in the directions parallel to the electrode plane, see Fig. 1) has no net effects on the electron-transfer processes. Thus, only the motion vertical to the electrode plane is effective and needs to be taken into account, while the molecule's motion on the x - y plane would be ignored.

In this work, we consider the simplest electrode reaction and electrochemical technique, i.e., the single-electron one-step process and the chronoamperometry. Considering the basic single electron transfer reaction



The O molecules were assumed to participate in the electron-transfer reaction only and the interaction between redox molecules was not considered in this study. Thus, in this simulation, each molecule's behavior was simulated individually and then brought together to account for the whole electrode process. The electrode potential was so high that electron-transfer reaction occurred at every collision between the O molecule and the electrode. The product R was set to leave immediately and have no influence on the electrode process.

Another important thing in the simulation is the choice of the parameters of time step τ and length step δ . In a general sense, these values should be consistent with realistic length and time scales of a redox molecule's random walk in the solution. Recently, the random walk algorithm was used to interpret the role of collisional encounters between the redox molecule and electrode in determining the kinetic limit in steady-state voltammetric measurements [39]. In that paper, a new way of the determination of τ and δ was described. In this study, these two parameters were set in a similar way and summarized as follows. First, the definition of molecule velocity v (units of centimeters/second) in computer simulation was

$$v = \bar{\delta} / \tau \quad (2)$$

where $\bar{\delta}$ is the average step length of the molecule's movements. Then, consider the Einstein relationship

$$D = \bar{\delta}^2 / 2\tau \quad (3)$$

where D is the diffusion coefficient (units of square centimeters/second). Combining Eqs. 2 and 3 provides expressions of mean step lengths

$$\bar{\delta} = 2D/v \quad (4)$$

Generally, values of D are readily measured. The root mean square velocity for hydrated ions in aqueous solution is $\sim 10^4$ cm/s, which is proved by both a theoretical estimate [40] and an experimental determination [41]. So given the typical value for $D = 5 \times 10^{-6}$ cm²/s in aqueous solution, we can estimate that the mean step length $\bar{\delta} \approx 0.01$ nm, which corresponds to τ is in about 0.1 ps. Although these values of τ and δ seem extremely small compared to previous computer simulation, these are consistent with realistic length and time scales that we can anticipate for the random motion of a redox molecule through the solution [39]. These values for τ and δ were adopted in the simulation below.

When the simulation is initiated, the molecule leaves its initial position and starts wandering in the confined cylindrical space, the typical trajectory of single-molecule diffusion is achieved similar to that presented by White’s paper [39]. Current vs time characteristics were recorded for a series simulation with molecules from 10 to 10^6 , with the same cell volume and electrode surface area. All the simulations were performed on a personal computer with a 3.4-GHz Pentium 4 CPU and 512 Mb of RAM by using a MatLab 7.0 software. Generally, the calculation time for one case is from several seconds to more than 100 h depending on the number of the molecules.

To compare these simulation results with theory based on macroscopic statistical theory, we adopted the current-time equation of the thin layer cell in the situation of diffusion control [38]

$$i(t) = \frac{4nFAD_0C_0^*}{l} \sum_{m=1}^{\infty} \exp\left[\frac{-(2m-1)^2\pi^2D_0t}{l^2}\right], \quad (5)$$

where n is the number of electrons transferred per molecule, F is Faraday’s constant, A is the effective area of the electrode, D_0 is the diffusion coefficient of the O molecules, C_0^* is its initial concentration, and l is the thickness of the thin layer cell. The corresponding $i \sim t$ curve of Eq. 5 for $m < 11$ was superimposed on each of the simulation results for comparison.

Results and discussion

The influence of the number of redox molecules

First, we want to address the relationship between the measured current and the number of the redox molecules. To make the $i \sim t$ curve more straightforward, the units of current were chosen so that one electron transfer event during one time interval τ gives a current $I_{\text{norm}} = 1$. The results for different number of molecules were presented in Fig. 2 with a fixed temporal resolution $\tau = 0.1$ ps.

As shown in Fig. 2, for only a few molecules, the current was totally quantized and discrete (Fig. 2a,b). The simultaneous collision of two or more molecules to electrode came in sight when addressed one thousand molecules (Fig. 2c). In the meantime, the quantized nature of a single molecule gradually became blurry along with the increase of simultaneous collision probability. When the molecules’ number goes up to 10^6 , the behavior of individual molecules cannot be distinguished easily (Fig. 2f). The transition of the electrochemical signal from totally discrete to more continuous was clearly shown in Fig. 2.

As a comparison, the results based on Eq. 5 were superimposed on every simulation result (Fig. 2) from which we can see that the two results get closer and closer apparently as the number of molecules increased. When the molecules’ number goes to 10^6 , the misfit between these two results can hardly be observed (Fig. 2f). Obviously, the individuality and quantized character of a system with a small number of molecules cannot be revealed by the continuum theory. These observations are consistent with previous simulations of dendrimer molecules by Amatore and co-workers [34].

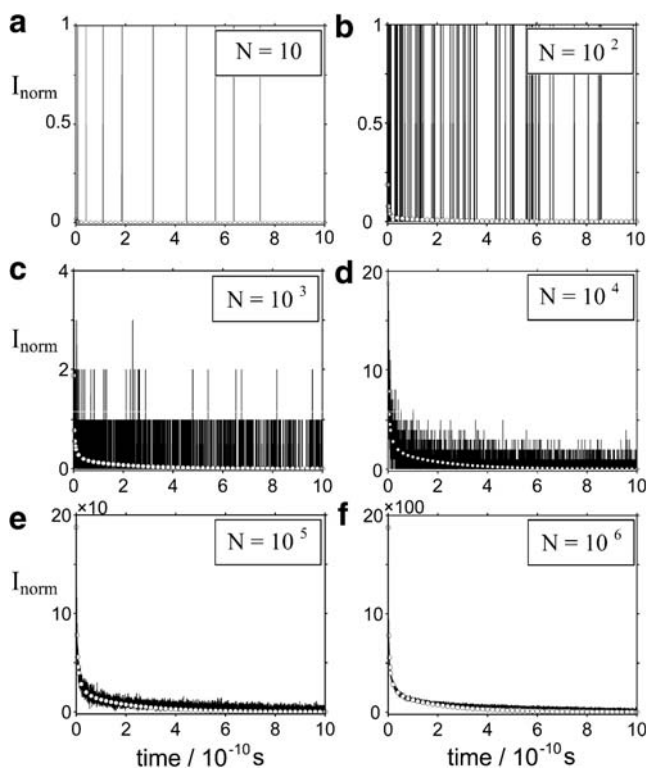


Fig. 2 The normalized current (I_{norm}) vs time as the molecule numbers N varies from 10 to 10^6 (see also the legend in a–f). The solid lines are the simulation results. And the circles filled in white are the results computed by Eq. 5

How to characterize the system exhibited with some uncertainty

From Fig. 2, it could also be observed that for a system with only a limited number of individuals, the result of every simulation is not always reproducible because of the prominence of individual behaviors. There are some difficulties in understanding the characteristics of such system, as it showed some uncertain and unpredictable behavior. However, it is not enough and not reasonable to explain these behaviors just by simple stochastic effects. In the following, we will suppose a routine to quantify the feature of such system, by using a correlation analysis.

According to discrete signal analysis, the similarity of two signals was usually expressed by correlation coefficient, which is defined as [42]

$$\rho = \frac{\sum_{n=1}^N x(n)y(n)}{\sqrt{\sum_{n=1}^N x(n)^2 \sum_{n=1}^N y(n)^2}} \quad (6)$$

where $x(n)$ and $y(n)$ are the discrete expressions of two signals, N is the length of the signal, here corresponding to the total simulation time. When $x(n) = y(n)$, $\rho = 1$. And when $x(n)$ and $y(n)$ are irrelevant completely, $\rho = 0$. When the two signals have similarity to some extent, $|\rho|$ is in between 0 and 1. With this discrete signal analysis method and taking the results shown in Fig. 2 as two series digital signals, the corresponding correlation coefficients can be evaluated and used to quantify the difference between the results based on the two different methods.

The results based on Eq. 5 imply that the system's behavior can be described by the classical statistical theory. So, the difference to the results based on Eq. 5 reflected the departure from the statistical character. In this work, this deviation was expressed by $Q_E = 1 - \rho$, i.e., a parameter of a system's quantized extent; here ρ is the correlation coefficient of the simulation results and the results of Eq. 5. Obviously, for the conventional electrochemical system with almost infinite molecules, the value of Q_E would be 0. As the molecules' number decreases, the quantized characteristic would be more prominent, the value of the Q_E should increase and its maximum limit was 1. The relationship between Q_E and the number of molecules is shown in Fig. 3a (curve 1). As expected, as the number of the molecules increased, the value of Q_E decreased from about 1 to 0, which indicated that it would be a parameter useful in describing such systems.

However, to be observed experimentally, this result would require a perfect electrochemical instrumentation so that the apparatus did not introduce any distortion itself. As it seems impossible for any electrochemical instruments to achieve this time scale (10^{-13} s) nowadays, the influence of the sampling interval on the value of Q_E was also studied

here. Figure 3a (curve 2 to curve 7) showed that when the sampling times become longer, the system's quantized characteristic would be blurred, i.e., the value of Q_E would decrease. However, it can be seen that a similar anti-S shape was obtained under every sampling interval considered. Although these data are not enough, the tendency shown in Fig. 3a means that Q_E can be used as a characteristic parameter to describe a system with a limited number of molecules at least qualitatively.

From a physical point of view, a system's quantized extent reflected the specific weight of the total "anti-homogenization" moments during the whole measuring interval. Generally, the thermal motion of the molecules would lead to the homogenization of a mixture; the same occurs in electrochemical solution during the electrolysis process. This would be true when considering a conventional macrosystem with redox molecules in molar level or the measuring time is long enough. However, for a system with only a limited number of molecules, as the stochastic behavior of individuals became

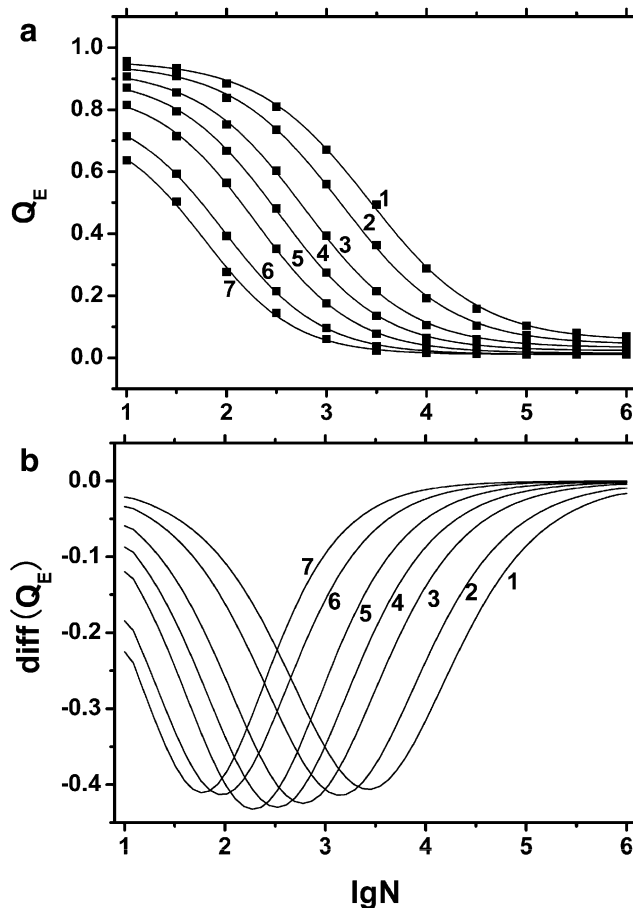


Fig. 3 **a** The relationship of the quantized extent (Q_E) and the number of the molecules of the system (in a logarithmic scale). From line 1 to line 7, the temporal resolution are 0.1, 0.2, 0.5, 1, 2, 5, and 10 ps, respectively. The *solid squares* are the average values of five parallel computations. The *solid lines* are the results of sigmoidal fit. **b** The derivative of the fitting results shown in **a**

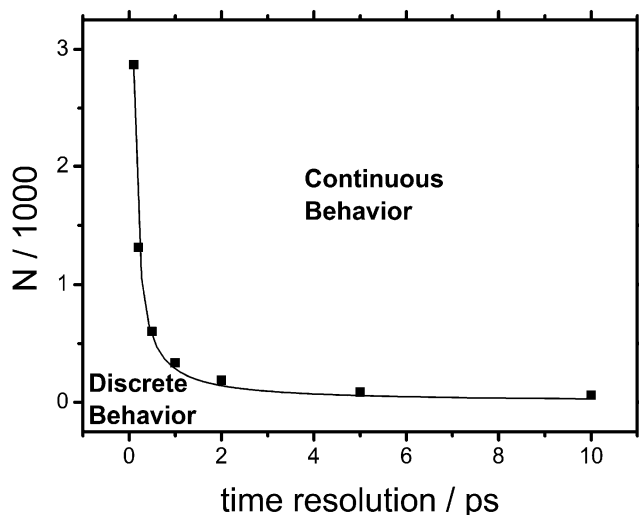


Fig. 4 The transition of the system's behavior from continuous to discrete, as the sampling times change from 0.1 to 10 ps. The *solid squares* correspond to the minima points in Fig. 3b. The *solid line* is the fitting result from Eq. 7

more dominant, the mass transfer would not always be proportional to the concentration gradient (maybe it is hard to define a concentration gradient when related to a system with only a few molecules; this term is used here in its conventional meaning); some kind of accelerating or decelerating phenomenon would be observed at a certain moment. In recent years, a great effort has been made to monitor concentration profiles within the diffusion layer created by an active interface, by using an ultramicroelectrode probe [43–46], SECM [47–49], and sometimes combined with spectrographic techniques [50, 51]. As excellent accuracy and resolution has been achieved by these works, one may expect the detection of these moments when the abnormal phenomenon occurred during the diffusion process. The value of Q_E reflected the specific weight of these moments during the whole measuring time, and the extent of the accelerating and decelerating respect to normal diffusion process at each moment were naturally included by this routine.

The anti-S shape of the $Q_E \sim \lg N$ curve implies that maybe there is a critical point to indicate the transition from discrete to continuous behavior for the systems with limited number of molecules. The derivative of Fig. 3a is shown in Fig. 3b. A clear V-shape dependence of the $dQ_E/d(\lg N)$ on the logarithm of the number of molecules was obtained under each temporal resolution considered. The minima corresponding to the molecules' number from about 2,900 to 60 related to sampling times vary from 0.1 to 10 ps.

These critical values permitted to give a rough division between the continuous and the discrete behavior, as the molecules' number and the temporal resolution changed. In Fig. 4 the influences of these two factors are clearly shown. We observed the relationship between these two factors

could be fitted by the following equation:

$$N \times t = C, \quad (7)$$

where N is the number of molecules, and t is the sampling intervals, C is a constant. When $N \times t \geq C$, the macroscopical statistical features play a dominant role in the whole system's behavior, and for a sufficiently small value of $N \times t$, the dominance of discrete characteristic with respect to the statistical nature occurred eventually. The fitting results indicated $C \approx 283$ ps here. Although not quite clear about the physical meaning of this constant now, Eq. 7 did reflect the fact that the quantized extent of a system was determined by the combined effect of the size of the system and the temporal resolution of the measuring tools.

In this simulation, the cell volume and electrode size are considered to be fixed values, so the electron transfer events during the same time interval are determined only by the number of the redox molecules. In a general consideration, these factors such as electrode surface area would have a similar influence on the electron transfer events, and correspondingly, on the Q_E of a system. Thus, this strategy can be extended to a broader application.

Conclusions

By random walk simulation, using an electrochemical model, the system's behavior clearly presents the transition from discrete to continuous related to the number of the molecules. Assisting with correlation analysis, an identified parameter Q_E was proposed to describe systems with a limited number of molecules, which reflected the specific weight of accelerating or decelerating of diffusion moments during the whole measuring interval. By this quantitative expression, the rough division between the continuous and the discrete behavior was presented related to the number of molecules and the temporal resolution.

It should be noted that this is an idealized model in which the influence of the electric field, the volume of molecules themselves, the interaction between molecules, etc., were not considered. And also, this simulation ignores the background noise in the real world, which would never be achieved in actual measurements using instruments, especially in nanometer or single-molecule scales. All these factors are required to be considered in the following theoretical analysis toward single-molecule scales. The work aimed at the modifications and improvements of this method for further theoretical research is currently under way.

This paper emphasized the warrant and necessity of developing new theories toward the understanding of discrete behaviors of systems with a limited number of molecules, i.e., the underlying rules of the electrochemical signals, which seemed stochastic and noisy.

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